The Effect of Reorientation of the Fibre Orientation Distribution on Fibre Tracking

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Abstract. Diffusion weighted imaging (DWI) allows to delineate neural fibres, based on local, directional information of the diffusion of water. Due to its directional nature, the local information needs to be reoriented upon image transformation, in order to preserve correspondence to the anatomy. In this work, we show that reorientation of the fODF with preservation of volume fractions (PVF) affects both deterministic and probabilistic fibre tracking. We identify the main causes for this, and validate them on synthetic and real brain DWI data. The problem is not with the PVF reorientation itself, but rather with the fODF reconstruction, its use in fibre tracking, and the influence of the seeds.

1 Introduction

Diffusion weighted imaging (DWI) is a magnetic resonance imaging (MRI) modality that measures the diffusion of water in vivo, along the direction of a diffusion gradient. High angular resolution diffusion imaging (HARDI) applies many, evenly distributed gradient directions to obtain a spherical distribution of the diffusion process in every voxel. Under the assumption that the diffusion is hindered by the tissue structure, and axon myelination in particular, DWI can estimate the axon directions in every voxel in the image. The fibre orientation distribution function (fODF) [1–3] represents the probability of fibres in a certain direction, based on HARDI data and given a convolution kernel $K(\theta)$. The fODF is defined by

$$S(\theta, \phi) = K(\theta) * \text{fODF}(\theta, \phi),$$

where $S(\theta, \phi)$ is the measured HARDI signal and $*$ denotes spherical convolution. The fODF reconstruction from HARDI data is then referred to as spherical deconvolution. Descoteaux et al. [3] have established linear relations between the HARDI signal and the fODF, using a basis of real, symmetric, orthonormal spherical harmonics (SH).

Fibre tracking aims at reconstructing the white matter (WM) connections, by “walking” along the estimated fibre direction. Deterministic fibre tracking methods follow the peaks, i.e., the local maxima, of the fODF. Probabilistic
methods draw random samples from a probability distribution like the fODF, and hence allow to assess the sensitivity of the tracking to local errors [4].

Registration, i.e., spatial alignment of images, is a common requirement in medical image analysis, e.g., for comparing images of different patients. All registration methods aim to optimize a similarity measure in the parameter space of a spatial transformation. After the transformation, image resampling on a regular grid is often required. The directional nature of DWI data imposes an additional requirement on the transformation, known as reorientation [5–7]. As shown in Fig. 1, the local fibre directions in each image need to be corrected for the transformation, as to preserve the coherence in the underlying tissue structure. In this work, we investigate how reorientation can affect fibre tracking.

![Figure 1](image_url)

**Fig. 1.** The need for reorientation: A vertical shearing is applied to the fODF image on the left. Without reorientation (middle) the correspondence to the anatomical bundles is lost. Reorientation (right) corrects the fODFs for the transformation.

## 2 fODF Reorientation and Fibre Tracking

### 2.1 Preservation of volume fractions

The fODF represents the fraction of fibres in each direction. If the image transformation locally compresses the fibre distribution on the sphere, the amplitude of the fODF should increase, similar to a change-of-variables of a PDF. This principle is referred to as preservation of volume fractions (PVF), and is formally stated as

$$ f_{\text{ODF}}(\theta, \phi) \, d\Omega = f_{\text{ODF}}'(\theta', \phi') \, d\Omega' , $$

(2)

where $d\Omega = \sin \theta \, d\theta \, d\phi$ is a small surface patch on the unit sphere. The primed symbols represent the coordinates and the fODF after transformation. From this equation, Hong et al. [5] have derived that

$$ f_{\text{ODF}}'(\theta', \phi') = f_{\text{ODF}}(\theta, \phi) \frac{\sin \theta}{\sin \theta'} \frac{1}{|\det(J_{\Omega})|} , $$

(3)
where $J_{Q}$ is the Jacobian of the angular transformation from $(\theta, \phi)$ to $(\theta', \phi')$, which can be derived from the deformation field at every voxel. Raffelt et al. [6] have presented a computationally efficient method that models the fODF as a weighted sum of SH $\delta$-functions. Dhollander et al. [7] have translated this method to the signal space and added an isotropic volume fraction. The use of one of these PVF methods generally gives rise to a result such as the one in Fig. 1, where the angles between the peaks of the fODF can change upon reorientation.

### 2.2 Deterministic fibre tracking

In order to study the effect of PVF reorientation on deterministic fibre tracking, we must look at its influence on the local maxima of the fODF. We identify 2 effects that occur in theory, i.e., *lobe reshaping* and *lobe interference*, and one additional effect that occurs due to aliasing in the SH basis.

![Fig. 2. The effect of lobe reshaping and lobe interference on the local maxima.](image)

**Lobe reshaping** PVF reorientation will generally change the shape of the fODF, and consequently alter the position of the local maxima. To avoid interference from other lobes, we restrict our study to single-fibre voxels. In addition, we aim to avoid unnecessary mathematical complexity by limiting the study to 2-D functions in polar coordinates, which can best be thought of as a cross-section of the fODF in the plane of the transformation. We define $f_{\varTheta}(\theta)$ as the (2-D) fODF of the original fibre direction $\varTheta$, $f_{\varXi}(\xi)$ as the fODF of the transformed fibre direction $\varXi$, and $g: \varTheta \to \varXi: \theta \mapsto g(\theta)$ as a monotonously increasing function that defines the local reorientation of the fODF. Equation (3) then becomes

$$f_{\varXi}(\xi) = \frac{dg^{-1}(\xi)}{d\xi} f_{\varTheta}(g^{-1}(\xi)) .$$

(4)
Let \( \theta^* \) be the maximum of \( f_{\theta}(\theta) \), i.e., \( \frac{df_{\theta}(\theta^*)}{d\theta} = 0 \) and \( \frac{d^2 f_{\theta}(\theta^*)}{d\theta^2} < 0 \). The derivative of (4) at the transformed maximum \( \xi^* = g(\theta^*) \) is then given by

\[
\frac{df_{\xi}(\xi^*)}{d\xi} = f_{\theta}(\theta^*) \frac{d^2 g^{-1}(\xi^*)}{d\xi^2},
\]

In general, this expression is not equal to 0, which proves that the peak of the transformed fODF does not correspond to the transformed peak of the original fODF. This is illustrated in Fig. 2b. In the special case where \( g \) is a pure rotation, the second derivative of \( g \) is 0 and \( \xi^* \) is a local maximum of \( f_{\xi} \). Hence, for a rigid rotation, the maxima of the transformed fODF correspond to the transformed maxima of the original fODF, as shown in Fig. 2.

**Lobe interference** As a second effect of reorientation, the lobes of the fODF in a voxel with crossing fibres can move towards or away from each other. Each lobe inevitably has a certain width, for several reasons. (i) The bandwidth of the fODF reconstruction is constrained by the spatial distribution of the HARDI samples in \( q \)-space. More samples increase the angular resolution and hence allow for narrower lobes. (ii) The noise regularization of the various fODF reconstruction techniques, whether by low-pass filtering, constrained SD or the finite order of the SH-basis, puts an additional constraint on the bandwidth. (iii) The true underlying fibre structure in one voxel is expected to have a certain spread as well, especially due to the partial volume effect. As a result of their width, the lobes can interfere with each other, and thereby influence the location of the maxima. This effect is illustrated in the right panel of Fig. 2. Note that lobe interference occurs as a general side effect of fODF reconstruction, even without reorientation. However, in its ability to change the local angular fibre density, fODF reorientation can increase or decrease the effect of interference.

**Aliasing in the SH basis** The SH basis can be thought of as the analogy of a Fourier series on a sphere. By this analogy, the SH \( \delta \)-function does not have an exact representation in a basis of finite order, but will rather have side lobes due to aliasing. In Raffelt’s method, a mixture of \( \delta \)-functions is fitted to the fODF, in such a way that the net effect of the side lobes is zero. However, when the \( \delta \)-functions are reoriented, the side lobes start to interfere in a complex pattern, and the net effect will no longer be zero. This can influence the peaks of the fODF and consequently fibre tracking.

### 2.3 Probabilistic fibre tracking

Probabilistic fibre tracks are generated step-by-step, by drawing random samples from the fODF. As opposed to the deterministic tracking, it is difficult to compare tracks based on correspondences, as the property of having one track per seed is lost. Instead, we can look at the statistical distribution of all generated fibre tracks before and after transformation. Ideally, the distribution of the tracks in
the transformed image should match the transformed distribution of the tracks in the original (untransformed) image.

At the local scale, the fODF is defined as the probability distribution of the direction of all tracks within a single voxel. Reorientation with preservation of volume fractions ensures, by definition, that the transformed fODF is equivalent to the distribution of the direction of the transformed tracks (within the limits of partial volume effects). This suggests that the statistical distribution of the tracks in the transformed image is equivalent to the distribution of transformed tracks of the original image.

However, in the above reasoning, it is implicitly assumed that the distribution of tracks, generated by probabilistic fibre tracking, matches the distribution of the true fibres that generate the fODF. While, in theory, this is indeed the goal of fibre tracking, in practice, it is generally not the case for several reasons. First of all, the reconstructed fODF does not entirely correspond to the true directional fibre density, for reasons associated with partial voluming and aliasing. Secondly, and more importantly, the topological density of the generated tractography will generally differ from the true fibre density. While the number of fibres that cross a volume element in white matter are a physiological property of the tissue, the number of generated tracks strongly depends on the distribution of the seeds. Moreover, if seeds are uniformly distributed in the original and the transformed image, the seeding density of the transformed original tracks differs as well.

3 Experimental Set-up

3.1 DWI data

**Synthetic image** We have created a synthetic image of a perpendicular crossing using a multi-tensor model. Horizontal and vertical diffusion tensors with eigenvalues $\lambda_1 = 0.0018\text{mm}^2/\text{s}$ and $\lambda_2 = \lambda_3 = 0.0006\text{mm}^2/\text{s}$ are created and converted to signals in $q$-space using 75 equally distributed gradient directions and $b = 3000\text{s/mm}^2$. At the crossing, we calculate the average of both reconstructed signals. The background is assumed to be isotropic, and is generated from a spherical diffusion tensor with eigenvalues $\lambda_1 = \lambda_2 = \lambda_3 = 0.002\text{mm}^2/\text{s}$. The voxel size is isotropic and equal to 1 mm.

**Real brain image** DWI data was obtained from one healthy volunteer, using a Siemens 3T scanner at an isotropic voxel size of 2.5 mm. The $S_0$ image was acquired as the average of 10 regular T2-weighted images ($b = 0\text{s/mm}^2$). In addition, 75 diffusion-weighted images $S(b, g)$ were recorded at $b = 2800\text{s/mm}^2$.

3.2 fODF estimation

The fibre orientation in each voxel, represented by the fODF, is estimated with constrained spherical deconvolution (CSD) [2] in an SH-basis of order 6. We use
the software package MRtrix\textsuperscript{4} [4]. The diffusion kernel is estimated directly from the DWI data, by aligning and averaging the signal profiles within a mask of single fibre voxels, retrieved as the voxels with FA $> 0.7$ [1, 2, 4].

### 3.3 Tractography algorithm

Fibre tracking is performed using MRtrix, with a standard Euler stepping method. In deterministic tracking, the direction of the next step is determined by of the fODF peak closest to the current direction. In probabilistic tracking, the next step is a random sample, drawn from the fODF using rejection sampling [4]. The step size is set to 0.2 mm. The fODF in each step is retrieved from trilinear interpolation on the SH coefficients. Tracks are terminated if the amplitude of the fODF peak falls below a threshold, set to 0.1, or if the local radius of curvature is smaller than 1 mm. Seeds are either distributed uniformly in a spherical ROI, or within a (full brain) mask, specified by the user.

### 3.4 Image transformation and reorientation

The synthetic image is submitted to a shearing along the $y$-axis, i.e., $F = [1 0 ; k 1]$, where the parameter $k = \tan \alpha$ is determined by the shearing angle $\alpha$. Different values for $\alpha$ are evaluated, ranging from $10^\circ$ to $60^\circ$. The transformation and resampling are performed in MATLAB\textsuperscript{5} on the raw DWI data. Reorientation is done on the signal values as well, using the method of Dhollander et al. [7] with 1000 evenly distributed alpha functions ($\delta$-functions translated to signal space).

The real brain image is submitted to a non-linear transformation. The deformation field is obtained from registration to another subject, recorded under similar conditions, using the registration algorithm described in [8].

### 3.5 Distance measure

We will use the current distance $D_{cd}$ [9, 10] to compare the full set of tracks in the transformed image to the transformed tracks of the original image, for different harmonic orders. The current distance between 2 sets of tracks $\mathcal{A} = \{A_1, A_2, \ldots, A_n\}$ and $\mathcal{B} = \{B_1, B_2, \ldots, B_m\}$, where each track $A_k$ and $B_l$ is a sequence of points $a_i$ and $b_j$, is defined as

$$D_{cd}(\mathcal{A}, \mathcal{B}) = \kappa(\mathcal{A}, \mathcal{A}) + \kappa(\mathcal{B}, \mathcal{B}) - 2 \kappa(\mathcal{A}, \mathcal{B}),$$

(6)

where $\kappa(\mathcal{A}, \mathcal{B})$ represents the similarity between $\mathcal{A}$ and $\mathcal{B}$ [10]. This similarity function is defined as

$$\kappa(\mathcal{A}, \mathcal{B}) = \sum_{A_k \in \mathcal{A}} \sum_{B_l \in \mathcal{B}} \sum_{a_i \in A_k} \sum_{b_j \in B_l} G_\sigma(||b_j - a_i||) (\delta_1^a a_i \cdot \delta_1^b b_j),$$

(7)

\textsuperscript{4} Freely available at http://www.nitrc.org/projects/mrtrix/ (GNU GPL)

\textsuperscript{5} The MathWorks Inc., Natick, MA (http://www.mathworks.com/products/matlab/)
where $G_\sigma(x)$ is a Gaussian kernel with standard deviation $\sigma$ (equal to the voxel size in our experiments), and the operator $\delta_1^1$ denotes the first order central difference, i.e., the tangent vector of the track in that point. As such, the current distance takes both the distance between points and the difference in direction into account. The Gaussian kernel ensures that the relative weight of points on distant tracks decreases.

4 Results

4.1 Synthetic data

The fODF reconstruction of the synthetic image before and after shearing is shown in Figs. 3 and 4. Deterministic fibre tracking is initiated from a spherical seeding region of radius 0.2 mm at the centre of the crossing. The resulting tracks are shown in red in Fig. 3a. These tracks are submitted to the image transformation, and then compared to the outcome of tracking on the transformed (and reoriented) fODFs, shown in green in Fig. 3b–f. For shearing angles under 30° the green tracks are straight lines. There is a clear deflection with respect to the transformed original tracks in red. The angular difference between the red and green tracks at the crossing is not stronger than in the single fibre voxels in the distal areas. This suggests that lobe interference has little effect if the shearing angle is small with respect to the angle of the crossing. For larger shearing angles, as shown in Figs. 3e–f, the effect of lobe interference gains importance, up to the point where both lobes merge and only one fibre direction is detected.

The result of probabilistic fibre tracking in the same set-up, and initiated from a single seed point at the centre of the crossing, is shown in Fig. 4. The overall distribution of the green tracks deflects from the distribution of the red tracks in much the same way as for deterministic tracking, because the fODFs have been reoriented identically. For large shearing angles, as shown in Fig. 4f, probabilistic tracking does succeed to recover some of the tracks in the horizontal (now displaced) fibre bundle, unlike deterministic tracking.

4.2 Full brain tracking

In the second experiment, we submit the real brain image to the non-linear transformation and reorient the signal values using PVF in the signal space. A full-brain tractography is then generated using 50 000 equally distributed seed points within a brain mask, both in the original image and in the transformed image. As in the previous experiment, we transform the original tracks to the new image and compare the result (red) to the tracks of the transformed image (green). The current distance $D_{cd}$ between both track sets is reported in Table 1, both for deterministic and probabilistic fibre tracking and for varying harmonic order $L$. We observe a decrease of the current distance for increasing harmonic order. Furthermore, the current distance at a particular harmonic order is smaller for probabilistic tracking than for deterministic tracking.
Fig. 3. Results of deterministic tracking on synthetic data. (a) Original image of crossing fibres. The initial tracking is shown in red. (b)–(f) Image after vertical shearing. The original tracks are transformed using the same shearing and shown in red. The outcome of fibre tracking in the transformed image is shown in green. Yellow then indicates overlap between both track distributions.
Fig. 4. Results of probabilistic tracking on synthetic data. (a) Original image of crossing fibres. The initial tracking is shown in red. (b)–(f) Image after vertical shearing. The original tracks are transformed using the same shearing and shown in red. The outcome of fibre tracking in the transformed image is shown in green. Yellow then indicates overlap between both track distributions.
Table 1. Current distance $D_{cd}$.

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<th>Probabilistic</th>
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<td>$L = 6$</td>
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<td>$L = 10$</td>
<td>1.064</td>
<td>0.911</td>
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5 Discussion and Conclusion

The fODF is, by definition, the probability distribution of the fibre directions in a voxel, and hence contains the maximal available information for fibre tracking. However, fibre reconstruction is still limited by the partial volume effect of the discrete measurements. It is for example not possible to discriminate between crossing and kissing fibres on a voxel scale. Deterministic fODF-based fibre tracking methods determine the direction of tracking as the peak of the fODF, closest to the current direction of the track. It has been pointed out in literature that the peaks do not necessarily correspond to the exact fibre directions due to noise or interference from other track directions [1–3]. Probabilistic methods draw random samples from the fODF to guide each step of the tracking.

We have shown that reorientation with preservation of volume fractions (PVF) affects the local maxima of the fODF, and hence deterministic fibre tracking. The peaks of a reoriented fODF do not always correspond to the reoriented peaks of the original fODF. We have identified 2 effects, directly associated with the PVF, that can cause this error, i.e., lobe reshaping and lobe interference, and an additional effect that hampers the PVF and influences the peaks as well, i.e., aliasing in the SH basis. Lobe reshaping occurs when a non-rigid transformation induces an unequal amount of compression in the local fibre density on the sphere. By preservation of volume fractions, local compression of the fODF will rightly lead to an increase of the amplitude of the fODF. Consequently, unequal compression will cause an unequal rescaling of the fODF, which can result in a shift of the local maximum. Lobe interference is a general effect in fODF reconstruction, that occurs mainly at sharp fibre crossings. It is relevant in the context of reorientation as well, as a non-rigid transformation can change the angle between the fibre directions. We have shown that, if the transformation pushes two fibre directions closer to each other, lobe interference will cause an additional shift of the peaks of the fODF towards each other, up to the point where both peaks merge and only one fibre direction can be retrieved. Due to the combination of lobe reshaping and lobe interference, fibre tracking on the non-rigidly transformed fODFs will produce a different result than transforming the result of fibre tracking on the original fODFs. The problem is not with the reorientation or the idea of PVF, but rather with the principle of deterministic fibre tracking itself. The local maximum of the fODF is in fact the mode of a PDF, which is known to be unstable upon a change of variables.

In our experiment on synthetic data, we have noticed that the green tracks indeed differ from the red tracks. However, the deflection is not according to the
predicted lobe reshaping. This suggests that the PVF reorientation is hampered by the limited order of the SH basis, that causes side lobes in the $\delta$-functions in Raffelt’s method due to aliasing. Upon reorientation, these side lobes will interfere in a complex pattern, that can influence the shape of the main lobes and hence the local maxima and the direction of tracking. In a recent paper, Raffelt et al. [11] have suggested the use of apodised $\delta$-functions to avoid the influence of the side lobes.

During the course of our work, we have noticed that the reorientation methods of Raffelt et al. [6] and Dhollander et al. [7] suffer both from aliasing artefacts. Moreover, Raffelt et al. [11] report identical effects when using Hong’s method [5]. This is surprising, as Hong’s method is a direct implementation of PVF re-orientation, and in no way uses $\delta$-functions that can introduce side lobes. We therefore suspect that the aliasing artefacts might have a more profound cause, i.e., the sparsity of the reoriented samples or $\delta$-function directions. The unequal distribution of the samples might indeed introduce Gibbs ringing artefacts when fitted in a SH-basis. The apodised $\delta$-functions might then perform better because they are wider, rather than because of the reduced side lobes. This remark is made as a point of discussion, and should be the topic of further research.

Probabilistic fibre tracking, initiated from a single seed, is in principle not influenced by lobe reshaping and interference, but does suffer from aliasing artefacts in the SH basis. As a result, the distribution of tracks of the transformed image deflects from the transformed distribution of tracks of the original image. Additionally, if a large number of randomly distributed seeds is used, e.g., for full-brain tractography, the density of the generated tracks might be unfair with respect to the anatomy (e.g., longer tracks will have more seeds and hence a larger track density). Moreover, the seeding distribution will generally differ between the original and transformed images.

In real data, we have reported a decrease of the current distance between the full brain tractography after transformation and the transformed tractography of the untransformed image for increasing harmonic order. For higher orders of the SH basis, the lobes are less wide. As such, the effects of lobe reshaping and interference will be smaller, and the influence on the tracking will be reduced. For a given order, the current distance is smaller for probabilistic tracking than for deterministic tracking, which might suggest that probabilistic fibre tracking is less affected by the reorientation.

In the case of a global, rigid transformation, the reorientation is a pure rotation that will not affect the shape of the fODF in any way. The distribution of the seeds in the original and transformed images will be equal as well. As such, we expect that rigid registration has no effect on fibre tracking, apart from interpolation effects. In the case of non-rigid registration, it is better to do the fibre tracking on the untransformed image, i.e., before the registration, and submit the outcome to the transformation. In the case that fibre tracking needs to be done after registration, e.g., for comparison to an atlas, one has to be aware of the deflection of the tracks, caused by the registration.
In conclusion, based on theoretical considerations and experiments on synthetic and real HARDI data, we have shown that spatial transformations can influence the outcome of fibre tracking, both for deterministic and probabilistic methods.

References